PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

WAEP Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

A system of equations, where b is a real constant, is as follows:

$$\begin{aligned} x - y + 3z &= 11 \\ x + 2y + 2z &= 3 \\ 2x + by + 4z &= 8 \end{aligned}$$

(a) Solve the system when $b = 3$. (4 marks)

(b) Interpret the system of equations geometrically when $b = 4$. (2 marks)

See next page

Question 2 **(6 marks)**

Polynomial P is defined as $P(z) = z^4 - 4z^3 + 14z^2 - 36z + 45$.

(a) Show that $z - 3i$ is a factor of $P(z)$. (2 marks)

(b) Solve $P(z) = 0$, writing solutions in Cartesian form. (4 marks)

Question 3 **(6 marks)**

Functions f, g and h are defined as

$$f(x) = x + 3, \qquad g(x) = \sqrt{x}, \qquad h(x) = \frac{4}{2 - x}.$$

(a) Determine

(i) $h \circ g \circ f(6)$. (1 mark)

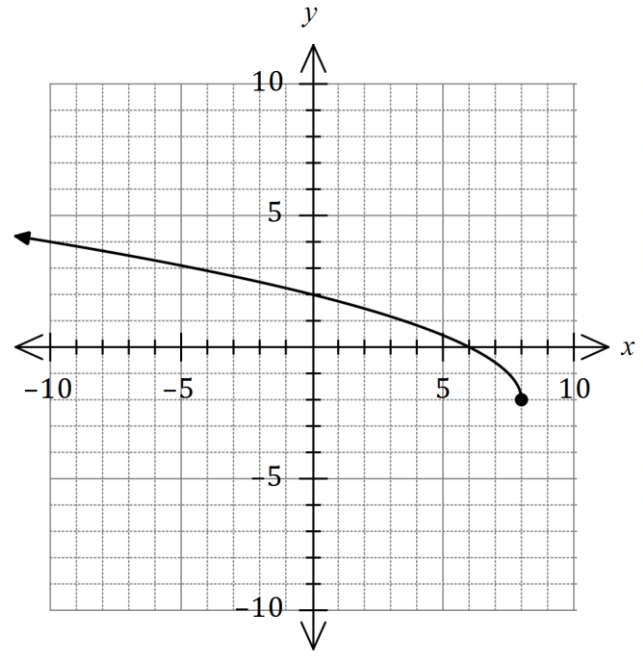
(ii) the defining rule for $h \circ g \circ f(x)$. (1 mark)

(b) Determine the domain of $h \circ g \circ f(x)$. (2 marks)

(c) Determine the range of $h \circ g \circ f(x)$. (2 marks)

Question 4 **(6 marks)**

The graph of $y = f(x)$ is shown below.



(a) Draw the graph of $y = f^{-1}(x)$ on the same axes. (3 marks)

(b) Given that $f(x) = \sqrt{16 - 2x} - 2$, determine the defining rule for $f^{-1}(x)$. (3 marks)

See next page

Question 5 **(8 marks)**

Points A, B, C and D have position vectors $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ and $\overrightarrow{OD} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$.

Note that $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$ given by $\theta = \cos^{-1}(\hat{\mathbf{u}} \cdot \hat{\mathbf{v}})$.

(a) Determine $\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$ and use the result to explain why A, B and C are collinear. (5 marks)

(b) Determine the Cartesian equation of the plane containing all four points. (3 marks)

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Question 6 **(7 marks)**

(a) Determine the complex cube roots of -1 in the form $a + bi$, where $a, b \in \mathbb{R}$. (3 marks)

(b) Let ω be a complex cube root of unity, $\text{Im}\,\omega \neq 0$, so that $\omega^3 - 1 = 0$.

(i) Show that $(\omega - 1)(\omega^2 + \omega + 1) = \omega^3 - 1$ and hence explain why $\omega^2 + \omega + 1 = 0$. (2 marks)

(ii) Simplify $(2 + 5\omega)(2 + 5\omega^2)$. (2 marks)

Question 7 **(6 marks)**

The equation of line L is

$$\frac{x-2}{2}=\frac{y+1}{-3}=\frac{z-1}{6}.$$

(a) Determine the vector equation of the line in the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$. (2 marks)

(b) The diameter of sphere S is the segment of line L between $x=2$ and $x=6$. Determine the equation of the sphere. (4 marks)

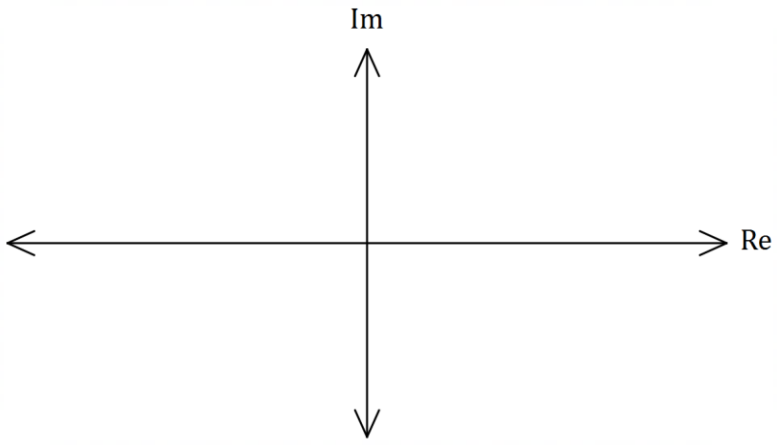
See next page

Question 8 **(7 marks)**

The locus L_1 of the complex number $z = x + iy$ has equation $|z - 6| = 2|z + 6|$.

(a) Show that L_1 is a circle with equation $x^2 + y^2 + 20x + 36 = 0$. (2 marks)

(b) Sketch L_1 on an Argand diagram. (2 marks)



Another locus L_2 has equation $w \cdot \bar{z} + \overline{w} \cdot z = 0$, where $w = 4 + 3i$.

(c) Show that L_2 is a tangent to L_1. (3 marks)

End of questions

Supplementary page

Question number: _____